Phase Lock Acquisition for Sampled Data PLLs Using the Sweep Technique

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Simulation results of the swept-acquisition performance of residual carrier phase-locked loops (PLLs) are reported. The loops investigated are sampled data counterparts of the continuous time type II and III loops currently in use in Deep Space Network receivers. It was found that sweep rates of 0.2 B_L^2 to 0.4 B_L^2 Hz/s can be used, depending on the loop parameters and loop signal-to-noise ratio (SNR), where B_L is the one-sided loop noise bandwidth. Type III loops are shown to be not as reliable as type II loops for acquisition using this technique, especially at low SNRs.

I. Introduction

It is well known that phase-locked loops (PLLs) are excellent tracking devices, but they can take an excessive amount of time to acquire when the loops are narrow-band compared to the input frequency offsets.

A popular method of improving the acquisition time is to sweep the center frequency of the local oscillator over the frequency range where the input signal is expected to be (Refs. 1–4). The sweep rate must be held within reasonable bounds; otherwise the PLL may not lock at all. Frazier and Page (Ref. 1) derived an empirical formula based on simulation for the maximum permissible sweep rate for a 0.90 probability of acquisition for noisy signals. Later Gardner (Ref. 4) suggested a more conservative limit for the allowable sweep rates based on practical experience.

These results are valid only for second-order, continuous time PLLs. To our knowledge similar documented information is lacking for third-order continuous time PLLs, or for sampled data versions of these two types of loops.

An Advanced Receiver is now being developed for the Deep Space Network. This receiver uses type II and type III sampled data loops (Ref. 5). The type III is required to achieve low steady state phase error with narrow bandwidths under conditions of high doppler rate, such as at Voyager Neptune encounter. Since the sweep technique is equivalent (as far as the PLL is concerned) to acquiring when frequency ramps are present in the input signal, it is interesting to determine if the type III is reliable for acquisition under these dynamic conditions. This would avoid the need to acquire first with a type II loop and subsequently switch to a type III loop.

We present computer simulations of the swept-acquisition technique. The PLLs involved are those pertinent to the Advanced Receiver, which have been previously discussed in Ref. 5. We present plots of probability of acquisition versus sweep rate with loop signal-to-noise ratio (SNR) as a parameter for type II and III loops.

We also examine in detail the impact of sampling rate on acquisition and pull-in behavior in general.

II. Description of Simulation

A detailed description of the baseband simulation model was reported in Ref. 6. The existing model only requires the addition of a frequency ramp term (opposite in sign to the frequency offset) to the input phase process. For the simulation runs, the local oscillator was swept 100 times, and the number of times that the loops acquired represented an estimate of the percentage of probability of acquisition. This procedure was repeated for different frequency rates and several combinations of loop parameters, SNR, and loop type.

Phase lock is declared once the phase error drops below 90 deg and stays there for at least ten times the reciprocal of the loop bandwidth. This definition is somewhat arbitrary, but we found it convenient to treat low SNR situations.

The initial frequency offset for every run was arbitrarily chosen as ten times the loop bandwidth, and the initial phase offset was uniformly distributed between $(-\pi, \pi)$. The mean and standard deviation of the acquisition times were recorded.

As in Ref. 6, the simulations were run for $B_L T = 0.02$, where B_L is the loop noise bandwidth, and T is the loop filter update time. This value is typical of the present breadboard implementation.

Other values of $B_L T$ were tested to determine their influence on pull-in behavior with, and without, frequency ramps present.

III. Simulation Results

Figures 1–3 contain the simulation results of probability of acquisition versus normalized sweep rate (where the normalized sweep rate is defined as the sweep rate in Hz/s, divided by B_L^2). In Tables 1 and 2 we include numerical values for the maximum permissible sweep rates for a 0.90 probability of acquisition. The mean times to acquire normalized by the bandwidth of the loop are shown, as well as the corresponding normalized standard deviations.

To compare the various loops, consider a loop SNR of 13 dB. For a 0.90 probability of acquisition Table 1 indicates that a type II loop with $r = 2$ as given in Ref. 5 can be swept at a normalized rate of 0.30. Table 2 indicates that a type II loop with $r = 4$ can be swept only at 0.25, and Fig. 3 indicates that a type III loop with $r = 4$ and $k = 0.25$ (Ref. 5) can be swept only at 0.17. For type II similar results hold for SNRs down to 7 dB, but type III loops with $r = 4$, $k = 0.25$ do not acquire reliably for loop SNRs below 13 dB. Thus, acquisition time is better for type II loops with $r = 2$ than for the other loops considered.

A type III loop with $r = 2$ was found to be unreliable for acquisition, even for small sweep rates, for all signal to noise ratios considered.

The uncertainty in the plotted points in Figs. 1–3 can be quantified in the following manner. Every simulation run represents a Bernoulli trial, since there are two possible outcomes. One is called a success (if the loop acquires) and the other one a failure (if the loop does not lock).

For a given sweep rate, denote the true probability of acquisition by p, and the probability of failure by q, where $p + q = 1$. The probability of r successes in any order, out of n simulation runs, follows a binomial distribution with mean np and variance npq.

Since n is large (100 in our case), the binomial distribution can be approximated by a Gaussian distribution. With this in mind, we obtain the 95% confidence intervals (following any standard text in statistics)

$$\bar{x} - \frac{1.96\sqrt{\bar{x}(1-\bar{x})}}{10} < p < \bar{x} + \frac{1.96\sqrt{\bar{x}(1-\bar{x})}}{10} \qquad (1)$$

where p is the true probability of acquisition for a given sweep rate, and $\bar{x}$ is the estimate of p obtained in the simulations.

These confidence intervals are indicated by bars in Figs. 1–3. In these figures, for the sake of clarity, bars were drawn only for one case of loop SNR. But, of course, similar bars can be drawn for other loop SNRs.

IV. Effects of Sampling Rate

Digital phase-locked loops possess certain distinctive features not encountered in analog phase-locked loops. For example, in theory, an analog PLL with a type II loop has an infinite pull-in range. This is not, in general, the case for digital PLLs. Early evidence supporting this statement may be found in Refs. 8–11. For this reason it is important to examine more carefully the effects introduced by possibly inadequate sampling rates (sampling rate is defined here as the reciprocal of the loop filter update time T).

In Fig. 4 we examine the sensitivity of the sweep technique with respect to $B_L T$. We plot the maximum normalized sweep rates that ensure a 0.90 probability of acquisition as a function of normalized bandwidth. This is done for a type II loop with $r = 2$ and without noise. A maximum normalized bandwidth of 0.08 is tolerable if the sweep technique is employed. Larger values are not allowed because stability problems arise (Ref. 5). (Stable loops can be designed for larger $B_L T$, but these loops are not considered here. See Ref. 7 for example.)

As stated before, Figs. 1–3 were obtained using $B_L T = 0.02$. For this value type II loops require a minimum normalized sweep rate of about 0.05 and type III a minimum of 0.16, otherwise the loops may not lock at all. Smaller values of $B_L T$ require smaller minimum sweep rates (approaching zero for a $B_L T = 0.01$, $r = 4$ type II when noise is not present), which agrees with intuition, since a sampled data loop with small $B_L T$ (<0.01) resembles a continuous time loop.

Computer simulations show that for a given bandwidth and sampling rate, these sampled data loops tolerate only a finite frequency offset. If this limit is exceeded, the loops become unstable and phase lock is never reached. In Fig. 5 we plot this maximum tolerable frequency offset as a function of bandwidth and sampling rate for a type II loop with $r = 4$ and no noise. The most surprising result is that the maximum allowed frequency offset varies almost linearly with the sampling rate.

More detailed analysis is required in the area of pull-in characteristics as a function of bandwidth, sampling rate, and initial phase conditions. This requires somewhat elaborate mathematical tools and graphical aids such as phase planes to study the convergence properties of the nonlinear difference equations describing the digital PLL operation. But this is considered to be outside the scope of our present discussion.

It is interesting to compare our results with empirical formulas reported in Refs. 1, 2, and 4. For a type II continuous time loop with $r = 2$ in a noiseless environment, Frazier (Ref. 1) and Lindsey (Ref. 2) suggest a frequency rate $R < 0.56\ B_L^2$, while Gardner (Ref. 4) suggests a conservative $R < 0.28\ B_L^2$. Our result, $R < 0.38\ B_L^2$, falls between these two values.

V. Effects of Initial Frequency Offsets

In the following discussion we will assume that the sampling rate is adequate such that frequency offsets much larger than $10\ B_L$ can be tolerated, as shown in Fig. 5.

The results presented are for initial frequency offsets of $10\ B_L$. The acquisition times are dominated by the sweep rate, and hence by the maximum sweep rate allowed for a given set of parameters. For type II loops the results can be extended to wider initial frequency offsets without modifying the maximum sweep rate. For type III loops, however, this is not valid for large initial offsets. Heuristically, this is explained as follows. When the initial offset is very large, the phase detector output is almost sinusoidal with nearly zero mean; therefore, the output of the loop filter (which is proportional to the frequency estimate) is approximately described by the integral of a Wiener process, whose variance grows as t^3, where t is time relative to the start of acquisition. For large initial offsets, t can be large and the rate of change of the loop filter output due to accumulated noise can be greater than the sweep rate. Lock may never occur either because the effective sweep rate is too high, or because the noise keeps the loop frequency from ever sweeping through the frequency of the incoming signal.

In conclusion, acquisition is more reliably achieved with type II loops.

Fast Fourier Transform (FFT) techniques can also be used in the acquisition mode to resolve the frequency uncertainty to a pre-determined range, typically a fraction of the loop bandwidth.

A fast acquisition scheme using FFT techniques has been added to the Advanced Receiver breadboard. Initial testing reveals that carrier phase lock can be achieved in 2 s with an initial frequency uncertainty of 350 Hz and a loop bandwidth of 4 Hz.

VI. Laboratory Results

Using a breadboard of the Advanced Receiver, laboratory measurements were taken to verify the results obtained by computer simulations. One hundred experiments were performed for three different cases, each of them with an initial frequency offset equal ten times the loop bandwidth. The bandwidth was set to 10 Hz and the sampling rate to 500 Hz.

Case 1 involved a type II loop with $r = 2$ and an estimated loop SNR of 13.3 dB. The loop was swept at a rate of 30.4 Hz/s, or, equivalently, a normalized rate of 0.304. It was observed that out of 100 trials, the loop attained phase lock 95 times. Extrapolation of the simulation results yields for the same SNR a probability of acquisition close to 0.92.

In case 2, a type II loop with $r = 4$ was employed. This time the estimated SNR was 12.8 dB and the normalized sweep rate 0.248. The loop acquired with probability 0.88. Extrapolation of the simulation results predict 0.87.

The last case consisted of a type III loop with $r = 4, k = 0.25$ and an estimated loop SNR of 13 dB. The normalized sweep rate was adjusted to 0.208. The measured probability of acquisition was only 0.42, in contrast with 0.82 predicted by the simulations.

It is believed that the discrepancy for type III loops can be attributed to small dc offsets in the output of the loop phase detector. In a type III loop, these dc offsets are accumulated in a double summer resulting in a rapid dc build-up, the net effect of which is to cause the filter output to run off in one direction. This has been observed in the breadboard loop by monitoring the NCO output. Thus, we conclude that in practical applications, undesired dc offsets may preclude the use of type III loops for acquisition.

VII. Empirical Formulas

Empirical formulas were obtained from the plots, relating maximum sweep rates, which gave a 0.90 probability of acquisition, as a function of loop SNR (ρ) and bandwidth. These are summarized as follows (assuming no dc offsets):

Type II, $r = 2$:

$$R < 0.38\, B_L^2 \left(1 - \frac{1}{\sqrt{\rho}}\right) \text{Hz/s}, \quad \rho > 7 \text{ dB} \tag{2}$$

Type II, $r = 4$:

$$R < 0.32\, B_L^2 \left(1 - \frac{1}{\sqrt{\rho}}\right) \text{Hz/s}, \quad \rho > 7 \text{ dB} \tag{3}$$

Type III, $r = 4, k = 0.25$:

$$R < 0.27\, B_L^2 \left(1 - \frac{1}{\sqrt{\rho}}\right) \text{Hz/s}, \quad \rho > 13 \text{ dB} \tag{4}$$

VIII. Conclusions

The simulations show that type II loops are more reliable for acquisition than type III loops when using the swept-acquisition technique. Type II loops can tolerate higher sweep rates and, hence, minimize acquisition time. Type III loops are not recommended for acquisition if this technique is employed, particularly if dc offsets are present.

We also explored in some detail the influence of sampling rate on pull-in behavior, pointing out fundamental differences between analog and digital PLLs.

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Table 1. Maximum permissible sweep rate for 0.90 probability of acquisition: corresponding normalized acquisition time and standard deviation for initial frequency offset of 10 B_L, type II, $r = 2$

SNR, dB	R_{90}/B_L^2	$B_L \bar{t}_{acq}$	$B_L \sigma_{acq}$
7	0.183	57.5	12.0
13	0.304	32.4	4.2
16	0.350	27.8	2.7
∞	0.380	24.4	0.9

Table 2. Maximum permissible sweep rate for 0.90 probability of acquisition: corresponding normalized acquisition time and standard deviation for initial frequency offset of 10 B_L, type II, $r = 4$

SNR, dB	R_{90}/B_L^2	$B_L \bar{t}_{acq}$	$B_L \sigma_{acq}$
7	0.145	67.6	14.7
13	0.248	41.0	6.4
16	0.280	36.3	5.9
∞	0.320	29.8	3.9

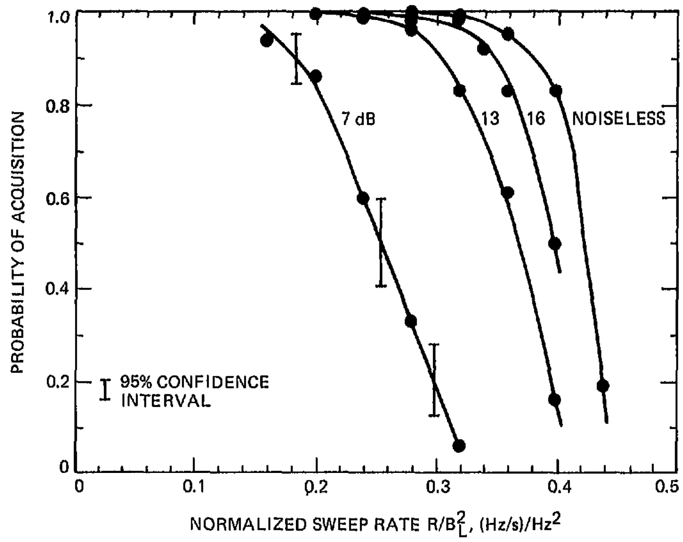

Fig. 1. Probability of acquisition vs normalized sweep rate with loop SNR as a parameter for a type II, r = 2 loop (initial frequency offset is ten times the loop bandwidth)

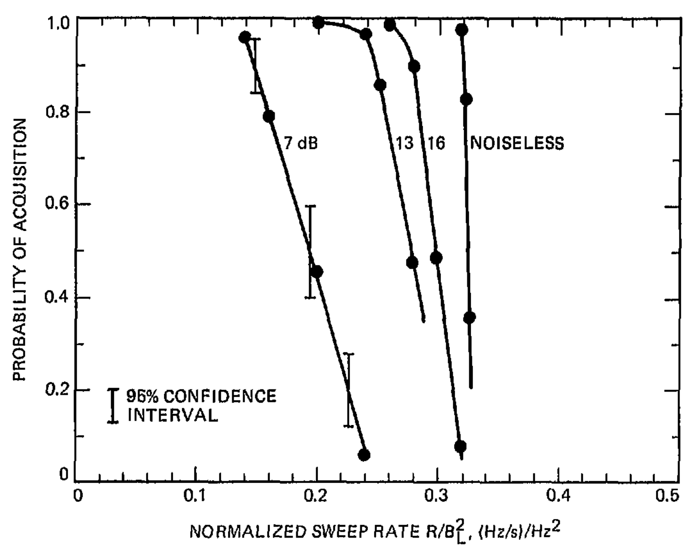

Fig. 2. Probability of acquisition vs normalized sweep rate with loop SNR as a parameter for a type II, r = 4 loop (initial frequency offset is ten times the loop bandwidth)

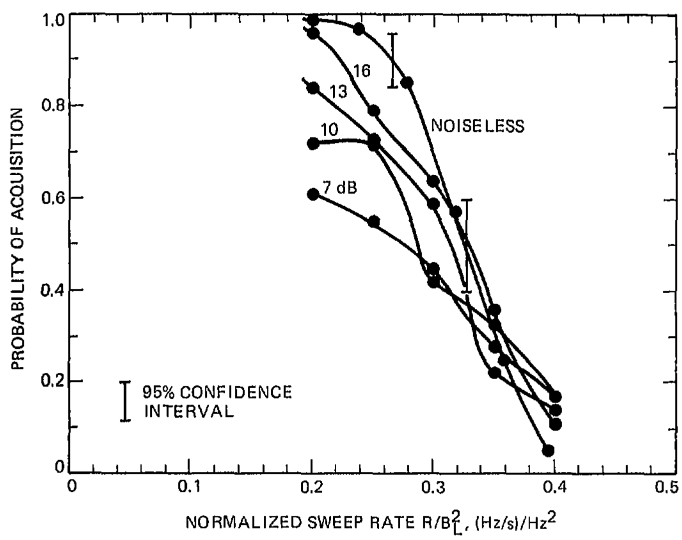

Fig. 3. Probability of acquisition vs normalized sweep rate with loop SNR as a parameter for a type III, r = 4, k = 0.25 loop (initial frequency offset is ten times the loop bandwidth)

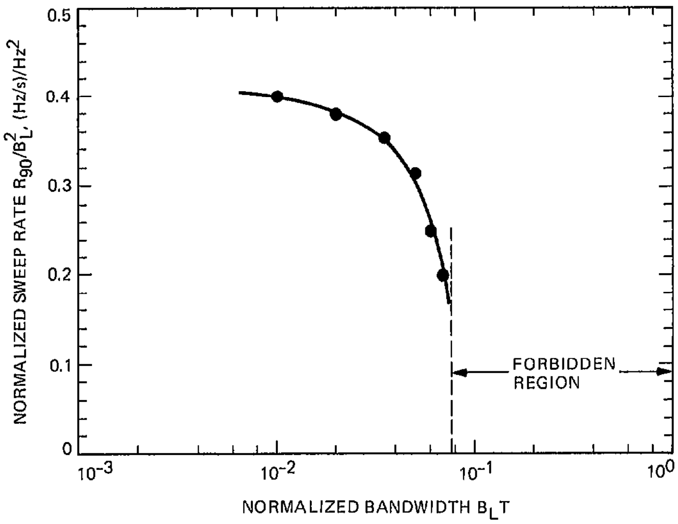

Fig. 4. Sensitivity of the sweep technique to B_LT for sweep rates that ensure 0.90 probability of acquisition: type II, r = 2

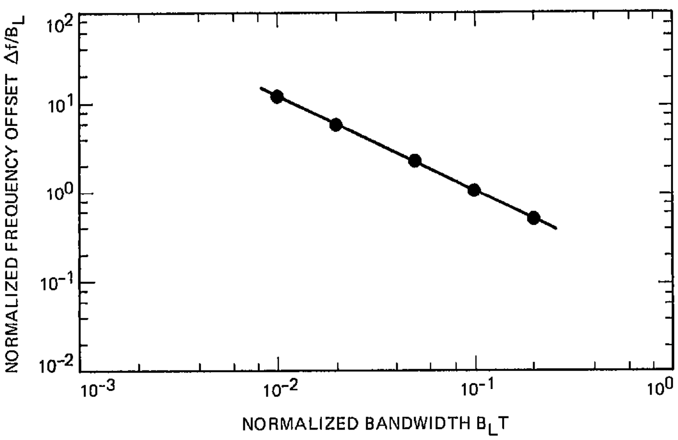

Fig. 5. Maximum initial frequency offset for which phase lock is guaranteed: type II, r = 4, SNR = ∞